

# The Breathing Modes of the $B = 2$ Skyrmion and the Spin-Orbit Interaction

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## Abstract

The coupling of the breathing and rotational modes of the skyrmion-skyrmion system leads to a nucleon-nucleon spin-orbit interaction of short range, as well as to spin-orbit potentials for the transitions  $NN \rightarrow N(1440)N$ ,  $NN \rightarrow NN(1440)$  and  $NN \rightarrow N(1440)N(1440)$ . The longest range behaviour of these spin-orbit potentials is calculated in closed form.

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The Skyrme model, and its generalizations, provide theoretically compact models for the baryons, which are able to describe most of their observed low energy properties to an accuracy of 30% or better [1, 2]. By quantization of the appropriate collective coordinates, the Skyrme model provides a natural and conceptually simple method for describing the rotational and vibrational degrees of freedom, that are revealed by the baryon spectrum. As an example the global adiabatic rotation of the skyrmion generates the spin- and isospin operators for the baryons [2]. Furthermore employment of the so called product ansatz for the  $B = 2$  skyrmion [3] makes it possible to quantize the motions of the individual skyrmions so as to generate a well defined nucleon-nucleon interaction operator for two spatially extended nucleons that is asymptotically exact [3, 4, 5].

To the important collective modes of the skyrmion also belong the breathing modes, which involve spherically symmetrical size vibrations. The breathing mode is responsible for the lowest even parity resonances of the nucleon and the  $\Delta_{33}$ : the  $N(1440)$  and  $\Delta(1600)$ , respectively [6, 7]. If the breathing mode is included as a quantum correction in the Hamiltonian it strongly affects the soliton profile [8]. The net effect of this additional breathing mode induced spin-orbit interaction in the  $NN$  system is small at large distances but at short range it appears – at least in the approximation below – to be large enough to change the sign of the isospin dependent spin-orbit interaction that is induced by the rotational modes [9, 10, 11].

We here consider the breathing modes for the two-skyrmion system treated in the product approximation, and show that their interplay with the rotational degrees of freedom leads to an effective spin-orbit interaction between two nucleons, concomitantly with a spin-orbit coupling between the nucleon nucleon ( $NN$ ) and the  $NN(1440)$  and  $N(1440)N(1440)$  channels.

The product approximation for the  $B = 2$  skyrmion, which is justified when the separation between the centers of the two skyrmions,  $\vec{R}_1(t)$  and  $\vec{R}_2(t)$  exceeds the skyrmion size, is [3]

$$U(\vec{r}, t; \vec{R}_1(t), \vec{R}_2(t)) = U_1(\vec{r} - \vec{R}_1(t); t) U_2(\vec{r} - \vec{R}_2(t); t) . \quad (1)$$

Here  $(\vec{r}, t)$  is the spacetime coordinate of the soliton field and  $U_i(\vec{r} - \vec{R}_i(t); t)$

is the adiabatically rotated hedgehog ansatz for the  $i$ -th skyrmion centered at  $\vec{R}_i(t)$ :

$$U_i(\vec{r}_i; t) = A_i(t)U_i^0(\vec{r}_i)A_i^\dagger(t), \quad U_i^0(\vec{r}_i) = \exp\{i\vec{\tau} \cdot \hat{r}_i F(r_i)\} . \quad (2)$$

Here we use the abbreviation  $\vec{r}_i \equiv \vec{r} - \vec{R}_i(t)$ , and denote the time dependent  $SU(2)$  rotation matrix for the  $i$ -th skyrmion  $A_i(t)$  and the chiral angle  $F(r)$  [2].

For a single ( $B = 1$ ) skyrmion the breathing mode coordinate  $\lambda(t)$  is introduced by the replacement of the static hedgehog field  $U^0(\vec{r})$  by the time dependent field  $U^0(\vec{r}, t) \equiv U^0(\lambda(t)\vec{r})$ . By this scale transformation the mass of the classical soliton becomes a function of the breathing coordinate with its absolute minimum at  $\lambda = 1$ . At the quantum level the strong coupling of the breathing and the rotational modes [7] shifts the minimum of the effective breathing potential  $V(\lambda)$  for a free nucleon to  $\lambda = \lambda_0 = 0.868$ . In the vicinity of this minimum potential  $V(\lambda)$  can be expanded as

$$V(\lambda_0 + \xi) \simeq V(\lambda_0) + \xi^2 V'' , \quad V(\lambda_0) = \frac{F_\pi}{e_S} \times 39.14 , \quad V'' = \frac{F_\pi}{e_S} \times 35.55. \quad (3)$$

For the parameter values  $F_\pi = 129 \text{ MeV}$  and  $e_S = 5.45$  used in ref.[2] one obtains the expectation values  $\langle 0|\xi^2|0 \rangle = 0.145$  and  $\langle 0|\xi^2|1 \rangle = 0.380$ , where  $|0 \rangle$  and  $|1 \rangle$  are the ground and the first excited breathing states of the  $B = 1$  skyrmion respectively.

For the  $B = 2$  skyrmion considered in the framework of the product ansatz (1) the breathing modes are introduced by the substitution  $U = U_1(\lambda_1 \vec{r}_1, t)U_2(\lambda_2 \vec{r}_2, t)$  [12]. The masses of the free solitons then become nonequal  $M(\lambda_1) \neq M(\lambda_2)$  with absolute minimum at  $\lambda_1 = \lambda_2 = \lambda_0$ . In this case the interaction between the two skyrmions with  $B = 1$  induces a coupling of the  $NN$ -system to  $NN(1440)$  and  $N(1440)N(1440)$  states, which gives rise to a repulsive central interaction between two nucleons [12].

Because the the product ansatz provides a reliable approximation only for the longest range behaviour of any component of the skyrmion-skyrmion interaction we shall here derive only the longest range component of the spin-orbit interaction, which is generated by the coupling of the vibrational and

rotational modes. For this it is sufficient to consider only the time derivatives in the quadratic term in the Lagrangian density of the model. The relevant part of the nucleon-nucleon interaction then takes the form [10]

$$V = -\frac{F_\pi^2}{8} \int d^3r \, Tr \left( U_1^\dagger(\vec{z}_1, t) \dot{U}_1(\vec{z}_1, t) \dot{U}_2(\vec{z}_2, t) U_2^\dagger(\vec{z}_2, t) \right) , \quad (4)$$

where  $\vec{z}_i = \lambda_i \vec{r}_i$  and the dots represent total time derivatives.

Using the projection formulae [5, 9]

$$\langle N_i' | A_i \tau_n j_m^{(i)} A_i^\dagger | N_i \rangle = -\delta_{nm} \langle N_i' | \frac{1}{6} \vec{\tau} \cdot \vec{\tau}^{(i)} + \frac{1}{2} | N_i \rangle , \quad (5)$$

$$\langle N_i' | A_i \tau_n A_i^\dagger | N_i \rangle = -\frac{1}{3} \langle N_i' | \sigma_n^{(i)} \vec{\tau} \cdot \vec{\tau}^{(i)} | N_i \rangle \quad (6)$$

one readily finds the explicit expressions:

$$\dot{U}_1^\dagger \dot{U}_1 = +\frac{i}{3} \vec{\tau} \cdot \vec{\tau}^{(1)} \left[ \frac{1}{I(\lambda_1)} + \frac{(\vec{\sigma}^{(1)} \cdot \vec{R} \times \dot{\vec{R}})(\vec{r}_1 \cdot \vec{R})}{2R^2 r_1^2} \right] \sin^2 F(z_1) + \dots , \quad (7)$$

$$\dot{U}_2^\dagger \dot{U}_2 = -\frac{i}{3} \vec{\tau} \cdot \vec{\tau}^{(2)} \left[ \frac{1}{I(\lambda_2)} + \frac{(\vec{\sigma}^{(2)} \cdot \vec{R} \times \dot{\vec{R}})(\vec{r}_2 \cdot \vec{R})}{2R^2 r_2^2} \right] \sin^2 F(z_2) + \dots , \quad (8)$$

where  $\vec{\sigma}^{(i)}$  and  $\vec{\tau}^{(i)}$  are the spin and isospin Pauli matrices for the  $i$ -th nucleon and  $I(\lambda_i)$  is its moment of inertia, the latter being a function of the breathing coordinate  $\lambda_i$  [7]. The dots indicate terms, which do not contribute to the spin-orbit force.

Substitution of the expressions (7) and (8) in (4) yields the spin-orbit interaction:

$$V_{LS} = (\vec{\tau}^{(1)} \cdot \vec{\tau}^{(2)}) \left\{ (\vec{\sigma}^{(1)} + \vec{\sigma}^{(2)}) \vec{l} V_+ + (\vec{\sigma}^{(1)} - \vec{\sigma}^{(2)}) \vec{l} V_- \right\} , \quad (9)$$

where the functions  $V_\pm$  are defined as

$$V_\pm = -\frac{F_\pi^2}{72MR^2} \int d^3r \left[ \frac{(\vec{r}_1 \cdot \vec{R})}{r_1^2 I(\lambda_2)} \pm \frac{(\vec{r}_2 \cdot \vec{R})}{r_2^2 I(\lambda_1)} \right] \sin^2 F(z_1) \sin^2 F(z_2). \quad (10)$$

Note the appearance of an antisymmetric spin-orbit interaction for the case when  $\lambda_1 \neq \lambda_2$ .

To estimate the long range behaviour of the potential we shall use the approximation where the field of one skyrmion is constant in the vicinity of the other one [5, 9]. This leads to the expressions

$$V_+ \simeq \frac{1}{12MR^2} \left[ v(\lambda_1 R) + v(\lambda_2 R) + \left( \frac{\lambda_2}{\lambda_1} \right)^3 u(\lambda_2 R) + \left( \frac{\lambda_1}{\lambda_2} \right)^3 u(\lambda_1 R) \right], \quad (11)$$

$$V_- \simeq \frac{1}{12MR^2} \left[ \left( \frac{\lambda_2}{\lambda_1} \right)^3 u(\lambda_2 R) - \left( \frac{\lambda_1}{\lambda_2} \right)^3 u(\lambda_1 R) \right], \quad (12)$$

where we have used the abbreviations

$$v(R) = -\sin^2 F(R), \quad u(R) = \frac{1}{3} R F'(R) \sin 2F(R). \quad (13)$$

In (11) we have used the quadratic approximation,

$$I(\lambda_i) = \frac{F_\pi^2}{6\lambda_i^3} \int d^3r \sin^2 F(r), \quad (14)$$

for the moment of inertia [5].

Expansion of the potentials  $V_+$  and  $V_-$  in the vicinity of the minima at  $\lambda_1 \simeq \lambda_0$  and  $\lambda_2 \simeq \lambda_0$  gives

$$V_+ \simeq V_0 + (\xi_1 + \xi_2) V_1^+ + (\xi_1^2 + \xi_2^2) V_2^+ + \xi_1 \xi_2 V_3^+, \quad (15)$$

$$V_- \simeq (\xi_1 - \xi_2) V_1^- + (\xi_1^2 - \xi_2^2) V_2^-, \quad (16)$$

respectively. Here  $\xi_i = \lambda_i - \lambda_0$  and the potential functions  $V_j^\pm$  are defined as

$$V_0 = \frac{1}{6MR^2} \left( -\sin^2 F + \frac{1}{3} \tilde{R} F' s \right), \quad (17)$$

$$V_1^+ = \frac{1}{36MR} \left[ (-2F' + \tilde{R} F'') s + 2\tilde{R} F'^2 c \right], \quad (18)$$

$$V_2^+ = \frac{1}{36MR} \left[ (12F' + 4\tilde{R} F'' + \frac{1}{2} \tilde{R}^2 F''' - 2\tilde{R}^2 F'^3) s \right. \\ \left. + (4\tilde{R} F'^2 - \frac{3}{2} \tilde{R} F'' + 3\tilde{R}^2 F' F'') c \right], \quad (19)$$

$$V_3 = -\frac{1}{12MR} \left[ (8F' + 2\tilde{R}F'')_s + (4\tilde{R}F'^2)_c \right] , \quad (20)$$

$$V_1^- = -\frac{1}{36MR} \left[ (4F' + \tilde{R}F'')_s + 2\tilde{R}F'^2_c \right] , \quad (21)$$

$$V_2^- = -\frac{1}{36M} \left[ (F''' - \frac{1}{2}\tilde{R}F'''' + 2\tilde{R}F'^3)_s + (4F'^2 + \tilde{R}F'F'')_c \right] . \quad (22)$$

In these equations we have introduced the following notations:  $s \equiv \sin 2F(\tilde{R})$ ,  $c \equiv \cos 2F(\tilde{R})$ ,  $\tilde{R} \equiv \lambda_0 R$ ,  $F \equiv F(\tilde{R})$ ,  $F' \equiv dF(\tilde{R})/d\tilde{R}$  etc.

The antisymmetrical spin-orbit term only contributes to the transition potential between the  $NN$  and  $NN(1440)$  channels. Denoting the nucleon state  $|0\rangle$  and the lowest breathing state (the  $N(1440)$ )  $|1\rangle$  we consider the following potential components:  $V_{MN}^\pm \equiv \langle 00|V^\pm|MN\rangle$ , where  $M, N = 0, 1$ :

$$V_{00}^+ = V_0 + 2 \langle 0|\xi^2|0\rangle V_2^+ , \quad (23)$$

$$V_{10}^+ = V_{01}^+ = \langle 0|\xi^2|1\rangle V_1^+ , \quad (24)$$

$$V_{11}^+ = (\langle 0|\xi|1\rangle)^2 V_3 , \quad (25)$$

$$V_{10}^- = -V_{01}^- = \langle 0|\xi^2|1\rangle V_1^- . \quad (26)$$

Here  $V_{00}^+$  is then the diagonal spin-orbit interaction in the nucleon-nucleon channel. In the approximation where the breathing excitations are frozen ( $\lambda_1 = \lambda_2 = 1$ ) the antisymmetrical potential  $V_-$  vanishes and the expression (9) reduces to the result of ref.[10].

The potentials (23-26) are shown in Figs. 1–3. The result in Fig. 1 shows that the breathing modes do not lead to any appreciable net modification of the isospin dependent component of the nucleon-nucleon spin-orbit interaction at long distances beyond 1 fm, but that at short distances their effect is strong enough to imply a change of sign. The short range behaviour of the results in the figures should however be viewed as tentative in view of the limited reliability of the product approximation at short range, and the omission of the effect of the stabilizing term in the Lagrangian density in these results. The large effect of the breathing modes on the spin-orbit interaction at short distances is a reflection of the softness against radial vibrations of the skyrmions, which is most obvious in the large underprediction of the mass of the Roper resonance [6, 7].

The results for the spin orbit components of the transition potentials in Figs. 2 and 3 also reveal the short range of these interactions, as well as the very large effect of the breathing modes on the interactions at short range.

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### Figure Captions

**Figure 1.** The isospin dependent nucleon-nucleon spin-orbit potential. The dotted line shows breathing mode correction  $V_0$  and full line the total spin-orbit interaction  $V_{00}^+$ . The potential is given in MeV; at  $R = 1.2 \text{ fm}$  the scale for the potential is changed.

**Figure 2.** The spin-orbit transition potentials  $V_{10}^+$  (dotted line) and  $V_{10}^-$  (full line). The potential is given in MeV; at  $R = 1.2 \text{ fm}$  the scale for the potential is changed.

**Figure 3.** The spin-orbit transition potential  $V_{11}^+$ . The potential is given in MeV.



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